Seismic Protection of Lead-Cooled Reactors

Modeling of Liquid Metals and Analysis of the Sloshing Effects

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SILER Project – Training Course
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**DOCUMENTS FOR REFERENCE**


- **Comprehensive treatise (950 pages)**
- **Author comes from Aerospace industry**
  - Book intended for mechanical & aeronautical engineers
  - Presents liquid sloshing effects on space vehicles, storage tanks, road vehicle tanks and ships, elevated water towers under ground motion

- **It covers:**
  - Linear & non-linear theory of liquid sloshing dynamics
  - Linear & non-linear interaction of sloshing with elastic containers
  - Spinning containers and microgravity sloshing
**DOCUMENTS FOR REFERENCE**

*Fluid Structure Interaction Effects on and Dynamic Response of Pressure Vessels and Tanks subjected to Dynamic Loading*, Steel Construction Institute, for the HSE-UK, Report RR527 (2007)

- Research report (200 pages)
- **Background is off-shore industry:**
  - Analysis methodologies, dynamic loads and simplified procedures for the determination of the response under earthquake, blast and ship impact.
- **It covers:**
  - State-of-the-art review
  - Determination of sloshing frequencies (baffled & un-baffled vessels)
  - Dynamic analysis / horizontal tanks
Fixed base rigid upright cylindrical/rectangular tanks
Extensively used in practice. First approximation to many practical problems:

- Incompressible and inviscid liquid
- Irrotational motion, satisfies Laplace’s equation
- Vertical velocity of liquid along base = ground velocity
- Radial velocities of liquid and tank wall are the same (rigid tank)
DOCUMENTS FOR REFERENCE


- Chapter 7, Seismic Response and Design of Liquid Storage Tanks, by A. S. Veletsos (110 pages)

- Introduces tank flexibility
  
  ◆ Dynamic characteristics of the tank-fluid system can be significantly different from that of a rigid tank.

  ◆ The so-called “flexible-impulsive” hydrodynamic component is introduced in the total response for design
**DOCUMENTS FOR REFERENCE**


- Based on ASCE (1984) as well as on results of more recent papers.
- Provides the most comprehensive simplified design procedures for fixed base cylindrical pressure vessels.
DOCUMENTS FOR REFERENCE


- *Paper describing the fundamentals of the method (32 pages).*

- *Provides the equations of motion, the treatment of viscosity and thermal conduction, the spatially-varying resolution, the types of kernels and the guidelines for computer implementation.*
**Physical Phenomenon**

- **Sloshing means any motion of the free liquid surface inside its container**
  
  - Basic problem involves the estimation of:
    
    - Hydrodynamic pressure distribution
    - Forces & Moments
    - Natural frequencies of the free-liquid surface

  ```
  ![Diagram of sloshing](taken from TID-7024)
  ```

  - **Hydrodynamic pressure** has two components:
    
    - **Impulsive**: directly proportional to the acceleration of the container, caused by the part of the fluid moving with the same container velocity
    - **Convective**: represents the “slosh” or free surface liquid motion.
**Physical Phenomenon**

- *Motion of the fluid can be caused by different dynamic excitations:*
  - **Earthquake**
    - The base of the container is subjected to ground or floor seismic motion
  - **Transportation**
    - Tanker ships, aircraft & spacecraft fuel tanks, road tankers…

Atmospheric storage tanks are very vulnerable to earthquakes
Physical Phenomenon

- Motion of the fluid can be caused by different dynamic excitations:
  - Earthquake
**Physical Phenomenon**

*Why to worry about sloshing in isolated NPPs?*

- Base isolation produces a shift in natural frequencies of buildings
  
  In NPPs, frequencies go from 2-8 Hz to 0.3-0.5 Hz

Sloshing frequencies for medium sized tanks are at the same frequency band: large amplification can occur.

If ELFR or MYRRHA reactors are base-isolated: sloshing of molten lead should be considered.

(taken from ASCE Primer on Seismic Isolation)
Simplified Methods

- Assumption of a “rigid” container (Housner, 1957)
  - Fixed base rigid upright cylindrical tank under seismic excitation
  - Fluid mechanics assumptions
    - Liquid is incompressible and inviscid
    - Motion is irrotational and satisfies Laplace’s equation
    - Structural and liquid motions remain linearly elastic
  - Boundary conditions
    - Vertical velocity of liquid along tank base is equal to ground velocity
    - Radial velocities of liquid and tank wall are the same
    - Free surface at top

(see equations in book by Ibrahim)
Simplified Methods

- Assumption of a “rigid” container (Housner, 1957)

  - Motion of liquid results in hydrodynamic pressures which can be split in two:
    
    “Impulsive” component: due to the part of the liquid which moves like a rigid-body, synchronously with the tank, as an added mass and is subject to the same acceleration levels as the tank.
    
    “Convective” component: due to the sloshing of the liquid at the free surface, since the other part of the fluid moves with respect to the tank walls.
**SIMPLIFIED METHODS**

- Assumption of a “rigid” container (Housner, 1957)

\[
F(0, t) = -m_0 \ddot{x} - \sum_{i=1}^{\infty} m_i (\dddot{x}_i + \dddot{x})
\]

- Impulsive force
- Convective forces

\[
m_0 = \text{impulsive mass}
\]

\[
m_i = \text{1st mode convective mass}
\]

\[
m_n = \text{n\textsuperscript{th} mode convective mass}
\]

\[
h_0 = \text{height at which impulsive mass attached to tank}
\]

\[
h_i = \text{height at which first mode convective mass attached to tank}
\]

\[
h_n = \text{height at which n\textsuperscript{th} mode convective mass attached to tank}
\]
SIMPLIFIED METHODS

- Assumption of a “rigid” container (Housner, 1957)

\[ \frac{m_i}{m_F} = \frac{8}{\pi^3} \frac{\tanh \left( (2i - 1) \frac{\pi h}{L} \right)}{(h/L)(2i - 1)^3} \]

Modal masses

Dynamic moment equilibrium gives heights of application of resultant forces

\[ \frac{h_i - h_G}{h - h_G} = \frac{1}{2} - \frac{\tanh \left( (2i - 1) \frac{\pi h}{2L} \right)}{(2i - 1) \frac{\pi h}{2L}} \]

CoG gives height of application of impulsive force

\[ \frac{h_0 - h_G}{h - h_G} = \frac{m_F}{m_0} \sum_i \frac{m_i (h_i - h_G)}{m_F (h - h_G)} \]
Simplified Methods

- Assumption of a “rigid” container (Housner, 1957)

Expressions by Graham & Housner are widely used in seismic design of tanks:

\[
\frac{m_0}{m_F} = \frac{\tanh\left(\sqrt{3}\frac{L/2}{h}\right)}{\sqrt{3}\frac{L/2}{h}}
\]

\[
\frac{h_0}{h} = \frac{1}{8} \left[\frac{4\sqrt{3}\frac{L/2}{h}}{\tanh\left(\sqrt{3}\frac{L/2}{h}\right)} - 1\right]
\]

\[
\frac{m_1}{m_F} = \frac{\tanh\left(1.58\frac{h}{L/2}\right)}{1.9\frac{h}{L/2}}
\]

\[
\frac{h_1}{h} = 1 - \left[\frac{\cosh\left(1.58\frac{h}{L/2}\right) - 1}{1.58\frac{h}{L/2}\sinh\left(1.58\frac{h}{L/2}\right)}\right]
\]
**Simplified Methods**

- **Assumption of a “rigid” container (Housner, 1957)**

Sloshing frequencies (Hz):

\[
 f_i = \frac{1}{2\pi} \sqrt{\frac{\lambda_i}{R}} \frac{g}{\lambda_i} \tanh\left(\frac{\lambda_i h}{R}\right)
\]

- \(g\) acceleration due to gravity
- \(R\) tank radius
- \(h\) height of fluid

\(\lambda_1 = 1.841\) \(\lambda_2 = 5.331\) \(\lambda_3 = 8.536\) \(\lambda_4 = 11.706\)

(vary with tank dimensions and are generally independent of liquid height, except for very shallow depths)
**Simplified Methods**

- Assumption of a “rigid” container (Housner, 1957)

Sloshing pressures along height (first 2 modes) for various $\gamma = h/R$ ratios

Maximum convective pressure: at $z = h$
- $r = R$

$$p_{c_{\text{max}}} = 0.837 \rho R \ Sa \ \cos \theta$$

where $Sa$ is the spectral acceleration at the fundamental sloshing frequency and damping ratio (about 0.5%)
Simplified Methods

- Assumption of a “rigid” container (Housner, 1957)

Maximum vertical displacement of fluid surface:

\[ d_{\text{max}} = 0.837 R \; Sa \]

where \( Sa \) is the spectral acceleration at the fundamental sloshing frequency and damping ratio (about 0.5%) expressed in \( g \).
Simplified Methods

- Flexible containers (ASCE, 1987)
  - Flexibility of the tank can result in the dynamic characteristics of the tank-fluid system to be significantly different from those of a rigid tank.

    A third hydrodynamic component was introduced: “flexible-impulsive”

- The convective component (sloshing) is generally assumed to be decoupled from the effects of tank flexibility.

  Convective effects arise from sloshing motion of liquid, whereas impulsive effects are caused by lateral motion of the tank and participating liquid.

  There is a wide frequency gap between motions associated with convective and impulsive effects. This further weakens the coupling.

  Pressure contributions from sloshing can be computed using the same expressions as for the “rigid” tanks.
**Simplified Methods**

- *Flexible containers (ASCE, 1987)*

- Total impulsive (rigid + flexible) maximum pressure (function of $z$):

\[
p_{it\text{max}} = C_0' \rho h \ Sa \ cos\theta
\]

where $S_a$ is the spectral acceleration at the natural frequency and damping ratio of the flexible tank-fluid system.

$C_0'$ is a coefficient which depends on the height to radius ratio ($h/R$), the distance to bottom ($z$) and the assumed deflection configuration:

\[
C_0' = (R/h) \ C_o
\]

(see figure in next viewgraph)
**Simplified Methods**

- **Flexible containers (ASCE, 1987)**
  - Vertical distribution of impulsive hydrodynamic maximum pressures.

![Diagram showing the vertical distribution of pressures in different types of tanks: slender tank, squat tank, and top of fluid.](image)

- Slender tank
- Squat tank
- Top of fluid
- Tank bottom
Simplified Methods

- **Flexible containers (ASCE, 1987)**
  - Fundamental natural frequency for tank-fluid system (impulsive mode)

\[
f_i = \frac{C_I}{2\pi} \cdot \frac{1}{h} \sqrt{\frac{E}{\rho_s}} \quad C_I = C_W \sqrt{\frac{\rho_W}{\rho_l}}
\]

(Haroun & Housner, 1981)

where:
- \(E\) = Young’s modulus of the material of the tank
- \(\rho_s\) = Density of wall material
- \(\rho_W\) = Density of water
- \(\rho_l\) = Density of fluid
- \(h\) = total height of fluid
- \(R\) = radius of tank
- \(C_W\) = frequency parameter (see graph)
Simplified Methods

- Flexible containers (ASCE, 1987)
  - Fundamental natural frequency for tank-fluid system (impulsive mode)

(Haroun & Housner, 1981)
**Simplified Methods**

- *Flexible containers (ASCE, 1987)*
  - Fundamental natural frequency for tank-fluid system (impulsive mode)

\[
 f_i = \frac{1}{2R \ g(\gamma)} \sqrt{\frac{E \ s_{1/3}}{\rho \ h}}
\]  

**(EC-8)**

where:

- \( E \) = Young’s modulus of the material of the tank
- \( s_{1/3} \) = Thickness of wall at 1/3 height
- \( \rho \) = Density of fluid
- \( h \) = total height of fluid
- \( R \) = radius of tank
- \( \gamma \) = \( h/R \)

\[
g(\gamma) = 0.01675 \ \gamma^2 - 0.15 \ \gamma + 0.46
\]
Flexible containers (ASCE, 1987)

- The main difference between the response of a rigid and flexible tank has to do with the nature of the acceleration (impulsive) component:

  In a “rigid” tank, the response is proportional to the maximum ground acceleration.

  In a “flexible” tank, the response is governed by the spectral acceleration corresponding to the fundamental frequency of the tank-liquid vibration and associated damping ratio (about 2%).

- Effects of vertical ground acceleration are considered to be always impulsive.

  Pressures are computed from vertical ground accelerations and the “breathing” mode frequency of the tank-fluid system.
**Numerical Methods**

- **Only way to deal with complicated geometries and excitations**
  - Purpose remains the same
    - Hydrodynamic pressure distribution
    - Resultant moments and forces on vessel and internal components
    - Displacements at free surface

- **Very difficult numerical problem**
  - Highly non-linear
    - Numerical techniques should be very robust.
    - Requires large computer resources
  - Coupling with structural deformation makes it even more difficult
    - Usually the numerical approach for the structure is different from the one used for the fluid.
**NUMERICAL METHODS**

- **Solution of Navier-Stokes equations in the time domain combined with methods for free surface tracking**
  - **Lagrangian**: mesh follows fluid
    Computation of parameters at moving points. Large deformations of discrete elements make integration very difficult.
  - **Eulerian**: mesh is fixed in space
    Computation of parameters at points fixed in space. Good for guided flows but it penalizes free surface computations.
  - **Arbitrary Lagrangian-Eulerian**: mesh at boundaries follows fluid
    Boundaries are Lagrangian. Inner volumes are re-meshed periodically.
  - **Volume Of Fluid (VOF)**: Eulerian + color function for separating phases
  - **Smoothed Particle Hydrodynamics (SPH)**: Fluid = Set of particles
    “Continuum” parameters are obtained by integration on neighbour particles using “kernel” functions
KASHIWAZAKI-KARIWA NPP

SFP overflow during 2007 Niigataken Chuetsu-oki Earthquake

- CRIEPR (Japan)
- Own VOF-3D code (non-linear wave behaviour).
- 3 axis motion from seismic data.
- Similar overflows and wave shape as recorded during EQ.
Study of sloshing of liquid lead at ELSY reactor vessel

- NRG (Netherlands)
- OpenFOAM code (VOF)
- Simple harmonic horizontal velocity motion from seismic data FRS at 3 first natural frequencies
- Shape and load assessment

(First mode results)
FUKUSHIMA DAI-ICHI NPP

SFP overflow during 2011 Tohoku Earthquake

- Lancemore (Japan)
- LS-DYNA (ALE)
- 3 axis motion from seismic records
Assessment of Spent Fuel Pool overflows

- IDOM (Spain)
- DualSPHysics (SPH)
- Motion derived from Floor Response Spectra
- 2D and 3D simulations
- With and without vertical motion
- Target: assess loss of water in the pool
Methodology

- Target FRS for horizontal and vertical motions at SPF elevation
- Derivation of compatible acceleration time-histories
- Modelization: definition of geometry, fluid properties, calculation parameters and motion. Particle size 0.10 m for 2D and 0.25 m for 3D
- Calculation with a SPH analysis tool. Time step = 0.1 s.
- Computation of overflows in a virtual outer container.
ALMARAZ AND TRILLO NPP STRESS TESTS - 3

Free surface behaviour

- First natural sloshing mode governs in overflows
- Small water losses (< 1 m³)
- Small influence of vertical motion
- «Focus» effect at corners for diagonal waves. Corners focus kinetic energy.
Experiences in seismic sloshing prediction in NPP

TRILLO NPP - 1

Stress tests. Assessment of Essential Services Pond overflows

- IDOM (Spain)
- DualSPHysics (SPH)
- Motion derived from Ground Response Spectra
- 2D simulations
- Without vertical motion (low influence)
- Target: assess loss of water in the pond
TRILLO NPP - 1

Stress tests. Assessment of Essential Services Pond overflows

• No overflow expected
• Induced vortex near lateral embankments
**Numerical Methods - SPH**

- *Smoothed Particle Hydrodynamics - SPH*

  - Particle-based method for solving fluid motion equations
    - Invented by Lucy (1977) and Gingold-Monagahan (1977)
    - Initially, for Astrophysics
    - Extended to weakly compressible flows by Monaghan (1994)

  - Attractive features
    - Lagrangian character / Interaction with structures is made easier
    - Exact treatment of convection (mass conservation)
    - Precise definition of interphases
    - Easy introduction of complex Physics

  - Present applications
    - Multiphase flows / Viscous flows
    - Fluid-structure interaction
    - High-speed impact problems
    - Landslide generated waves
Numerical Methods - SPH

- Smoothed Particle Hydrodynamics - SPH
  
  - Software (most of them are open source)
    
    SPH-flow
    Fluids v.1
    ISPH
    SimPARTIX
    SPHysics / DualSPHysics
    GPUSPH
**Numerical Methods - SPH**

- *Smoothed Particle Hydrodynamics - SPH*

  - The fluid is divided into a set of discrete elements (particles)
    - Particles have a spatial distance ("smoothing length")
    - Particle properties are "smoothed" over this distance by a "kernel function"
Smoothed Particle Hydrodynamics - SPH

\[ A_s(r) = \sum_i m_i \frac{A_i}{\rho_i} W(r - r_i, h) \]

- **Kernel function**
- **Particle property**
- **Smoothed property at position** \( r \)
- **Particle mass and density**
- **Smoothing length**

Derivatives of \( A \) (e.g. pressure) are built from derivatives of the kernel function.
Sloshing in Liquid Metal Pools

- *Important liquid properties (assuming low viscosity)*
  - Natural frequencies (sloshing)
    Depend on geometry and on the ratio of surface tension to density
  - Wave height (sloshing)
    Depends just on geometry
  - Hydrodynamic pressures
    Directly proportional to density
**Density of lead**

\[ \rho(T) = (11367 - 1.1944 T) \frac{\text{kg}}{\text{m}^3} \]

(Data from «Handbook on Lead-bismuth Eutectic Alloy and Lead properties», OCDE-NEA, 2007, NEA no. 6195)
Sound velocity in molten lead

(Data from «Handbook on Lead-bismuth Eutectic Alloy and Lead properties», OCDE-NEA, 2007, NEA no. 6195)
5 – Sloshing in liquid metal pools

Physical Properties of Lead - 1

Dynamic viscosity in molten lead

(Data from «Handbook on Lead-bismuth Eutectic Alloy and Lead properties», OCDE-NEA, 2007, NEA no. 6195)
PHYSICAL PROPERTIES OF LEAD - 2

Surface tension of lead

\[ \sigma(T) = (0.519 - 0.000113 T) \frac{N}{m} \]

(Data from «Handbook on Lead-bismuth Eutectic Alloy and Lead properties», OCDE-NEA, 2007, NEA no. 6195)
**Physical Properties of Lead - 3**

Properties of molten lead and water

- Water at 20 °C.
- Molten lead at 480 °C (ELSY)

<table>
<thead>
<tr>
<th>Property</th>
<th>Water (kg/m³)</th>
<th>Molten lead (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density ρ</td>
<td>1000</td>
<td>10470</td>
</tr>
<tr>
<td>Sound velocity c (m/s)</td>
<td>1500</td>
<td>1740</td>
</tr>
<tr>
<td>Surface tension σ (N/m)</td>
<td>0.466</td>
<td>0.434</td>
</tr>
<tr>
<td>Dynamic viscosity μ (mPa s)</td>
<td>1,002</td>
<td>1.882</td>
</tr>
<tr>
<td>σ / ρ</td>
<td>4.7 (10^{-4})</td>
<td>0.4 (10^{-4})</td>
</tr>
</tbody>
</table>

Most relevant difference is in density.
Same acceleration will produce pressures ten times larger.
CONCLUDING REMARKS & RESEARCH WITHIN SILER

- Sloshing of molten lead in a LCR is a difficult design problem
  
  Current design formulae only give a very rough approximation.

  Numerical approaches are needed in order to obtain:
  - Hydrodynamic pressures
  - Resultant forces on the internals
  - Maximum height of fluid

- Influence of the vessel flexibility has to be investigated

  Similarly to the “flexible-impulsive” mode for tanks.
  It is not clear that the vessel can be considered “rigid”
  This introduces further complication: fluid-structure interaction
CONCLUDING REMARKS & RESEARCH WITHIN SILER

- Within the SILER project:
  
  PhD thesis
  
  “Modelling of fluid structure interaction in sloshing flows, with application to Lead Fast Reactors”
  
  Universidad Politécnica de Madrid
  Naval Architecture Department

Take an open code SPH program.
Solve the coupling with structural (FE)
Validate with experimental results
Application to ELSY & MYRRHA
CONCLUDING REMARKS & RESEARCH WITHIN SILER

- Within the SILER project:

  Sloshing.avi

Experimental facilities
CONCLUDING REMARKS & RESEARCH WITHIN SILER

Within the SILER project:

Cable layer ship. Rectangular tank with baffles. T=2sec. 4cm water level. 6º Roll Amplitude

Experimental facilities
Non-Coupled SPH

Sloshing3.avi
Sloshing4.avi